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Slip flow and heat transfer of a second grade fluid past a stretching sheet through a porous space

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Abstract

Analytical solutions of the equations of motion and energy of a second grade fluid for the developed flow over a stretching sheet with slip condition are presented. The electrically conducting fluid occupies the semi-infinite porous space. The non-linear partial differential equations and boundary conditions are reduced to a system of non-linear ordinary differential equations and boundary conditions by similarity transformations. Homotopy analysis method (HAM) is implemented to solve the reduced system. Graphs are plotted for various values of the emerging dimensionless parameters of the problem and discussed.

Keywords: Second grade fluid; Heat transfer; Slip condition; Stretching sheet; Porous space; Analytic solution; Magnetohydrodynamic fluid

1. Introduction

The studies of boundary layer flows of viscous and non-Newtonian fluids over a stretching surface have received much attention because of their extensive applications in the field of metallurgy and chemical engineering, for example, in the extrusion of polymer sheet from a dye or in the drawing of plastic films. Such investigations of magnetohydrodynamic (MHD) flows are very important industrially and have applications in different areas of research such as petroleum production and metallurgical processes. Fortunately, stretching flow problem is one of those rare problems in fluid dynamics for which an exact analytical solution has been found in the literature. This type of flow has been considered first time by Sakiadis [1]. Even more interesting is the fact that the problem still admits an analytical solution when several other aspects are taken into account, separately or jointly, such as suction at the sheet [2,3], presence of transverse magnetic field [4,5] viscoelasticity of the fluid [6–14] partial slip at the boundary [15] and mass and heat transfer [16–19].

The no-slip boundary condition is known as the central tenets of the Navier–Stokes theory. But there are situations wherein such condition is not appropriate. Especially, no-slip condition is inadequate for most non-Newtonian fluids. For example, polymer melts often exhibit macroscopic wall slip and that in general is governed by a non-linear and monotone relation between the slip velocity and traction. The fluids exhibiting boundary slip find applications in technology such as in the polishing of artificial heart valves and internal cavities. Navier [20] suggested a slip boundary condition in terms of linear shear stress. The study of heat transfer in porous space has applications in numerous areas such as thermal and insulating engineering, modelling of packed sphere beds, solar power collector, cooling of electronic system, ventilation of rooms, chemical catalytic reactors, grain storage devices, petroleum reservoirs and ground hydrology.

Therefore the present work has been undertaken in order to analyze the flow and heat transfer characteristics due to a stretching sheet with slip effects. An electrically conducting second grade fluid fills the porous space. The outline of the paper is as follows.

In the next section we present the problem formulation. Section 3 gives the analytic solutions for the outcoming

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problem of the velocity and temperature fields. Homotopy analysis method (HAM) has been used in obtaining the analytic solutions. HAM is a recent newly developed method that has been already implemented for several problems [21–40]. In Section 4 we discuss the convergence of the solutions. Section 5 consists of graphical results and discussion. Then in Section 6 we provide concluding remarks.

2. Mathematical formulation

We consider a steady two-dimensional flow of an incompressible second grade fluid saturated in a porous space past a stretching sheet at $y = 0$. The fluid fills the porous space above the sheet $y > 0$. The flow is caused because of the application of two equal and opposite forces along the stretching sheet with a linear velocity cx, $c > 0$, therefore the origin is kept fixed. The fluid adheres to the sheet partially and thus motion of the fluid exhibits the slip condition. A uniform transverse magnetic field of strength B_0 is applied. The induced magnetic field is neglected under the assumption of small magnetic Reynolds' number. Furthermore, heat transfer analysis is taken into account by assigning T_w and T_∞, the sheet temperature and the temperature of the ambient fluid, respectively. Following [13,15,41], the governing boundary layer equations in Cartesian coordinate system are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^3 u}{\partial x\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] - \frac{\sigma B_0^2}{\rho}u - \frac{\mu\phi}{k}u, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_1\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \alpha_1\left[\left(\frac{\partial u}{\partial y}\right)\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\right], \tag{3}$$

where u and v are velocity components in x and y directions, respectively, ν is the kinematic viscosity, ρ is the fluid density, ϕ and k are porosity and permeability of the porous space, respectively, σ is the electrical conductivity of fluid, c_p is the specific heat, T is the temperature, k_1 is thermal conductivity and α_1 ($\geqslant 0$) is the material parameter of second grade fluid.

For viscous fluid, the slip flow condition has been employed by Navier and then used in studies of fluid flow in rough and coated surfaces [42], and gas and liquid flow in microdevices [43]. Recently Andersson [44] and Ariel et al. [45] analyzed the slip effects on the flows of viscous fluid and elastico-viscous fluid, respectively. The slip flow conditions in these investigations have been defined in terms of the shear stress. Therefore, employing the similar procedure, the slip flow conditions for the problem under consideration are

$$u(x,0) - \beta_1\left[\frac{\partial u}{\partial y} + \frac{\alpha_1}{\mu}\left\{2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + v\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^2 u}{\partial x\partial y}\right\}\right]\Bigg|_{y=0} = cx, \quad u(x,\infty) = 0, \quad v(x,0) = 0 \tag{4}$$

and the relevant conditions of temperature are

$$T(x,0) = T_w, \quad T(x,\infty) = T_\infty, \tag{5}$$

in which β_1 is the slip parameter and c is the stretching constant. Note that the thermal slip effects are not considered here. Introducing the non-dimensional parameters and variables as follows:

$$\eta = \sqrt{\frac{a}{\nu}}y, \quad u = axf'(\eta), \quad v = -\sqrt{a\nu}f(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad M^2 = \frac{\sigma B_0^2}{\rho c}, \quad \lambda = \frac{\nu\phi}{kc}, \quad \alpha = \frac{\alpha_1 c}{\rho\nu},$$
$$\beta = \beta_1\rho\sqrt{c\nu}, \quad Pr = \frac{\mu c_p}{k_1}, \quad Ec = \frac{c^2x^2}{c_p(T_w - T_\infty)}, \tag{6}$$

where β is the non-dimensional slip parameter, M is Hartman number, λ is the non-dimensional porosity parameter, α is the second grade parameter, Pr is the Prandtl number and Ec is the Eckert number. The incompressibility condition (1) is satisfied automatically and Eqs. (2) and (3) become

$$f''' + ff'' - f'^2 - M^2f' - \lambda f' + \alpha(2f'f''' - f''^2 - ff^{\mathrm{iv}}) = 0, \tag{7}$$

$$\theta'' + Prf\theta' + PrEcf''^2 + \alpha PrEc(f'f''^2 - ff''f''') = 0. \tag{8}$$

Boundary conditions (4) and (5) are transformed into [44,45]

$$f(0) = 0, \quad f'(\infty) = 0,$$
$$f'(0) - \beta f''(1 + 3\alpha f')|_{\eta=0} = 1, \tag{9}$$
$$\theta(0) = 1, \quad \theta(\infty) = 0. \tag{10}$$

The non-linear system of Eqs. (7)–(10) is solved analytically by using HAM.

3. Solutions for velocity and temperature using HAM

It is straightforward to select

$$f_0(\eta) = \frac{1}{1+\beta}[1 - \exp(-\eta)], \tag{11}$$

$$\theta_0(\eta) = \exp(-\eta) \tag{12}$$

as the initial approximations of $f(\eta)$ and $\theta(\eta)$. Besides that we choose

$$\mathscr{L}_1(f) = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta}, \quad (13)$$

$$\mathscr{L}_2(f) = \frac{df}{d\eta} - \beta\frac{d^2 f}{d\eta^2}, \quad (14)$$

$$\mathscr{L}_3(f) = \frac{d^2 f}{d\eta^2} - f \quad (15)$$

as the auxiliary linear operators having the following properties:

$$\mathscr{L}_1[C_1 + C_2 e^{\eta} + C_3 e^{-\eta}] = 0, \quad (16)$$

$$\mathscr{L}_3[C_4 e^{\eta} + C_5 e^{-\eta}] = 0 \quad (17)$$

in which C_i, $(i = 1\text{–}5)$ are arbitrary constants. If p $(\in[0,1])$ and $\hbar_i$ $(i = 1\text{–}3)$ indicate the embedding and non-zero auxiliary parameters, respectively, then the zeroth-order deformation problems are

$$(1-p)\mathscr{L}_1[\hat{f}(\eta;p) - f_0(\eta)] = p\hbar_1 \mathscr{N}_1[\hat{f}(\eta;p)], \quad (18)$$

$$\hat{f}(0;p) = 0, \quad \hat{f}'(\infty;p) = 0,$$
$$(1-p)\mathscr{L}_2[\hat{f}(\eta;p) - f_0(\eta)] = p\hbar_1 \mathscr{N}_2[\hat{f}(\eta;p)] \quad \text{at } \eta = 0, \quad (19)$$

$$(1-p)\mathscr{L}_3[\hat{\theta}(\eta;p) - \theta_0(\eta)] = p\hbar_2 \mathscr{N}_3[\hat{\theta}(\eta;p), \hat{f}(\eta;p)], \quad (20)$$

$$\hat{\theta}(0;p) = 1, \quad \theta(\infty;p) = 0, \quad (21)$$

where

$$\begin{aligned}\mathscr{N}_1[\hat{f}(\eta;p)] &= \frac{\partial^3 \hat{f}(\eta;p)}{\partial\eta^3} + \hat{f}(\eta;p)\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2} - \left(\frac{\partial \hat{f}(\eta;p)}{\partial\eta}\right)^2 \\ &\quad - M^2 \frac{\partial \hat{f}(\eta;p)}{\partial\eta} - \lambda\frac{\partial \hat{f}(\eta;p)}{\partial\eta} \\ &\quad + \alpha\left[2\frac{\partial \hat{f}(\eta;p)}{\partial\eta}\frac{\partial^3 \hat{f}(\eta;p)}{\partial\eta^3} - \left(\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2}\right)^2 \right. \\ &\quad \left. - \hat{f}(\eta;p)\frac{\partial^{iv} \hat{f}(\eta;p)}{\partial\eta^{iv}}\right],\end{aligned}$$

$$\begin{aligned}\mathscr{N}_2[\hat{f}(\eta;p)] &= \frac{\partial \hat{f}(\eta;p)}{\partial\eta} \\ &\quad - \beta\left[\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2} + 3\alpha\frac{\partial \hat{f}(\eta;p)}{\partial\eta}\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2}\right] - 1,\end{aligned}$$

$$\begin{aligned}&\mathscr{N}_3[\hat{\theta}(\eta;p), \hat{f}(\eta;p)] \\ &= \frac{\partial^2 \hat{\theta}(\eta;p)}{\partial\eta^2} + Pr\hat{f}(\eta;p)\frac{\partial \hat{\theta}(\eta;p)}{\partial\eta} + PrEc\left(\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2}\right)^2 \\ &\quad + \alpha PrEc\left(\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2}\right)\left[\frac{\partial \hat{f}(\eta;p)}{\partial\eta}\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2} \right. \\ &\quad \left. - \hat{f}(\eta;p)\frac{\partial^3 \hat{f}(\eta;p)}{\partial\eta^3}\right].\end{aligned}$$

The problems at the mth-order can be written as

$$\mathscr{L}_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 \mathscr{R}_{1m}(\eta), \quad (22)$$

$$f_m(0) = f'_m(\infty) = 0, \quad \mathscr{L}_2[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 \mathscr{R}_{2m}(\eta) \quad \text{at } \eta = 0, \quad (23)$$

$$\mathscr{L}_3[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_2 \mathscr{R}_{3m}(\eta), \quad (24)$$

$$\theta_m(0) = \theta_m(\infty) = 0, \quad (25)$$

$$\begin{aligned}\mathscr{R}_{1m}(\eta) &= f'''_{m-1}(\eta) - M^2 f'_{m-1} - \lambda f'_{m-1} \\ &\quad + \sum_{k=0}^{m-1}\left[f_{m-1-k}f''_k - f'_{m-1-k}f'_k\right. \\ &\quad \left. + \alpha\left(2f'_{m-1-k}f'''_k - f''_{m-1-k}f''_k - f_{m-1-k}f^{iv}_k\right)\right],\end{aligned} \quad (26)$$

$$\begin{aligned}\mathscr{R}_{2m}(\eta) &= f'_{m-1}(\eta) - (1-\chi_m) \\ &\quad - \beta\left[f''_{m-1} + 3\alpha\sum_{k=0}^{m-1} f'_{m-1-k}f''_k\right] \quad \text{at } \eta = 0,\end{aligned} \quad (27)$$

$$\begin{aligned}\mathscr{R}_{3m}(\eta) &= \theta''_{m-1}(\eta) + Pr\sum_{k=0}^{m-1}\left[\theta'_{m-1-k}f_k + Ecf''_{m-1-k}f''_k\right. \\ &\quad \left. + \alpha Ec\sum_{l=0}^{k}(f'_{m-1-k}f''_{k-l}f''_l - f_{m-1-k}f''_{k-l}f'''_l)\right],\end{aligned} \quad (28)$$

$$\chi_m = \begin{vmatrix} 0, & m \leqslant 1, \\ 1, & m > 1. \end{vmatrix} \quad (29)$$

Using the symbolic software Mathematica, we solve Eqs. (22)–(25) up to first few order of approximations. The Eqs. (22)–(25) give us the following type of solutions:

$$\begin{aligned}f(\eta) &= \sum_{m=0}^{\infty} f_m(\eta) \\ &= \lim_{M\to\infty}\left[\sum_{m=0}^{M} a^0_{m,0} + \sum_{n=1}^{M+1} e^{-n\eta}\left(\sum_{m=n-1}^{M}\sum_{k=0}^{2m+1-n} a^k_{m,n}\eta^k\right)\right],\end{aligned} \quad (30)$$

$$\begin{aligned}\theta(\eta) &= \sum_{m=0}^{\infty} \theta_m(\eta) \\ &= \lim_{M\to\infty}\left[\sum_{m=0}^{M} b^0_{m,0} + \sum_{n=1}^{M+2} e^{-n\eta}\left(\sum_{m=n-1}^{M+1}\sum_{k=0}^{2m+1-n} b^k_{m,n}\eta^k\right)\right].\end{aligned} \quad (31)$$

In above solutions, the recurrence formulas for the coefficients $a^k_{m,n}$ and $b^k_{m,n}$ of $f_m(\eta)$ and $\theta_m(\eta)$ are obtained when $m \geqslant 1$, $0 \leqslant n \leqslant 2m+1$ and $0 \leqslant k \leqslant 2m+1-n$ as

$$\begin{aligned}a^0_{m,0} &= \chi_m\chi_{m+2}\chi_{2m+1}a^0_{m-1,0} - \frac{1}{1+\beta} \\ &\quad \times\left[\sum_{q=0}^{2m}\Delta^q_{m,1}(\mu^q_{1,1} - \mu^q_{1,0} - \beta(\mu^2_{1,1} - 2\mu^q_{1,1} + \mu^q_{1,1}))\right. \\ &\quad + \sum_{n=2}^{m+1}\sum_{q=1}^{2m+1-n}\Delta^q_{m,n}\left\{\mu^q_{n,1} - n\mu^q_{n,0} - \beta(\mu^q_{n,2} - 2n\mu^q_{1,1}\right. \\ &\quad \left.\left. + n^2\mu^q_{1,1})\right\}\right] + \frac{1}{1+\beta}\sum_{n=0}^{m+1}\Pi^0_{m,n} - \sum_{q=0}^{2m}\Delta^q_{m,1}\mu^q_{1,0} \\ &\quad - \sum_{n=2}^{m+1}\sum_{q=1}^{2m+1-n}\Delta^q_{m,n}\mu^q_{n,0},\end{aligned}$$

$$a^0_{m,0} = \chi_m\chi_{m+1}\chi_{2m}a^0_{m-1,1} + -\sum_{q=0}^{2m}\Delta^q_{m,1}\mu^q_{1,0} - \frac{1}{1+\beta}\sum_{n=0}^{m+1}\Pi^0_{m,n}$$
$$+\frac{1}{1+\beta}\left[\sum_{q=0}^{2m}\Delta^q_{m,1}(\mu^q_{1,1}-\mu^q_{1,0}) + \sum_{n=2}^{m+1}\sum_{q=1}^{2m+1-n}\Delta^q_{m,n}\{\mu^q_{n,1}-n\mu^q_{n,0}\} - \beta\left\{\sum_{q=0}^{2m}\Delta^q_{m,1}(\mu^2_{1,1}-2\mu^q_{1,1}+\mu^q_{1,1}) + \sum_{n=2}^{m+1}\sum_{q=1}^{2m+1-n}\Delta^q_{m,n}\{\mu^q_{n,2}-2n\mu^q_{1,1}+n^2\mu^q_{1,1}\}\right\}\right],$$

$$a^k_{m,0} = \chi_m\chi_{m+2}\chi_{2m+1-k}a^k_{m-1,0}, \quad 0\leqslant k\leqslant 2m+1,$$

$$a^k_{m,1} = \chi_m\chi_{m+1}\chi_{2m-k}a^k_{m-1,1} + \sum_{q=k-1}^{2m}\Delta^q_{m,1}\mu^q_{1,k}, \quad 1\leqslant k\leqslant 2m+1,$$

$$a^k_{m,n} = \chi_m\chi_{m+2-n}\chi_{2m+1-n-k}a^k_{m-1,n} + \sum_{q=k}^{2m+1-n}\Delta^q_{m,n}\mu^q_{n,k},$$
$$2\leqslant n\leqslant m+1,\ 0\leqslant k\leqslant 2m+1-n,$$

$$b^0_{m,0} = \chi_m\chi_{m+2}\chi_{2m+1}b^0_{m-1,1} - \sum_{n=2}^{m+2}\sum_{q=0}^{2m+1-n}\Gamma^q_{m,n}\mu1^q_{n,0},$$

$$b^k_{m,1} = \chi_m\chi_{m+2}\chi_{2m-k}b^k_{m-1,1} + \sum_{q=k-1}^{2m+1}\Gamma^q_{m,1}\mu1^q_{1,k},$$
$$1\leqslant k\leqslant 2m+1,$$

$$b^k_{m,n} = \chi_m\chi_{m+3-n}\chi_{2m+2-n-k}b^k_{m-1,n} + \sum_{q=k}^{(2m+1-n)}\Gamma^q_{m,n}\mu1^q_{n,k},$$
$$2\leqslant n\leqslant m+2,\ 0\leqslant k\leqslant 2m+1-n,$$

$$\mu^q_{1,k} = \sum_{p=0}^{q+1-k}\frac{q!}{k!2^{q+2-k-p}}, \quad q\geqslant 0,\ 1\leqslant k\leqslant q+1, \tag{32}$$

$$\mu^q_{n,k} = \sum_{r=0}^{q-k}\sum_{p=0}^{q-k-r}\frac{-q!}{k!(n-1)^{q+1-k-r-p}n^{r+1}(n+1)^{p+1}},$$
$$q\geqslant 0,\ 1\leqslant k\leqslant q,\ n\geqslant 2, \tag{33}$$

$$\mu1^q_{1,k} = \sum_{p=0}^{2q+2}\frac{(-1)^{2q+1-2k-p}q!}{2^{q+2-k}k!}, \quad q\geqslant 0,\ 0\leqslant k\leqslant q+1, \tag{34}$$

$$\mu1^q_{n,k} = \sum_{p=0}^{q-k}\frac{q!}{k!(n+1)^{p+1}(n-1)^{q+1-k-p}},$$
$$q\geqslant 0,\ 1\leqslant k\leqslant q,\ n\geqslant 2, \tag{35}$$

$$\Delta^q_{m,n} = \hbar_1\Big[\chi_{m+2-n}\chi_{2m-n-q+1}(e^q_{m-1,n}-(M^2+\lambda)c^q_{m-1,n}) + \chi_{2m-n-q+2}(\alpha^q_{m,n}-\varrho^q_{m,n}) + \chi_{2m-n-q+2}\alpha(2\gamma^q_{m,n}-\delta^q_{m,n}-\omega^q_{m,n})\Big], \tag{36}$$

$$\Pi^q_{m,n} = \hbar_2\Big[\chi_{m+2-n}\chi_{2m-n-q+1}c^q_{m-1,n} - \chi_{2-n}\chi_{2-n-q}(1+\beta) \times\Big(1-\chi_m)a^q_{m-1,n} - \beta\{\chi_{m+2-n}\chi_{2m-n-q+1}d^q_{m-1,n}+\alpha\beta^q_{m,n}\}\Big], \tag{37}$$

$$\Gamma^q_{m,n} = \hbar_3\Big[\chi_{m+3-n}\chi_{2m-n-q+1}g^q_{m-1,n} + Pr\chi_{2m-n-q+3}\Lambda^q_{m,n} + PrEc\chi_{2m-n-q+2}\alpha^q_{m,n} + \alpha PrEc(\vartheta^q_{m,n}-\varphi^q_{m,n})\Big], \tag{38}$$

$$\alpha^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} d^i_{k,j}a^{q-i}_{m-1-k,n-j},$$

$$\varrho^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} c^i_{k,j}c^{q-i}_{m-1-k,n-j},$$

$$\gamma^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} a^i_{k,j}f^{q-i}_{m-1-k,n-j},$$

$$\delta^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} a^i_{k,j}f^{q-i}_{m-1-k,n-j},$$

$$\omega^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} a^i_{k,j}f^{q-i}_{m-1-k,n-j},$$

$$\beta^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} a^i_{k,j}d^{q-i}_{m-1-k,n-j},$$

$$\pi^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} a^i_{k,j}e^{q-i}_{m-1-k,n-j},$$

$$\Lambda^q_{m,n} = \sum_{k=0}^{m-1}\sum_{j=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{i=\max\{0,q-2m+2k+1+n-j\}}^{\min\{q,2k+1-j\}} a^i_{k,j}g^{q-i}_{m-1-k,n-j},$$

$$\vartheta^q_{m,n} = \sum_{k=0}^{m-1}\sum_{r=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{s=\max\{0,q-2m+2k+n-r\}}^{\min\{q,2k+1-r\}}\sum_{j=\max\{0,r-k+l-1\}}^{\min\{r,l+1\}} \times\sum_{i=\max\{0,s-2k+2l-1+r-j\}}^{\min\{s,2l+1-j\}} d^i_{l,j}d^{s-i}_{k-l,r-j}c^{q-s}_{m-1-k,n-r},$$

$$\varphi^q_{m,n} = \sum_{k=0}^{m-1}\sum_{r=\max\{0,n-m+k\}}^{\min\{n,k+1\}}\sum_{s=\max\{0,q-2m+2k+n-r\}}^{\min\{q,2k+1-r\}}\sum_{j=\max\{0,r-k+l-1\}}^{\min\{r,l+1\}} \times\sum_{i=\max\{0,s-2k+2l-1+r-j\}}^{\min\{s,2l+1-j\}} f^i_{l,j}d^{s-i}_{k-l,r-j}a^{q-s}_{m-1-k,n-r},$$

$$c^k_{m,n} = (k+1)a^{k+1}_{m,n} - na^k_{m,n}, \quad d^k_{m,n} = (k+1)c^{k+1}_{m,n} - nc^k_{m,n},$$
$$e^k_{m,n} = (k+1)d^{k+1}_{m,n} - nd^k_{m,n}, \quad f^k_{m,n} = (k+1)e^{k+1}_{m,n} - ne^k_{m,n},$$
$$g^k_{m,n} = (k+1)b^{k+1}_{m,n} - nb^k_{m,n}, \quad h^k_{m,n} = (k+1)g^{k+1}_{m,n} - ng^k_{m,n},$$
$$a^0_{0,0} = \frac{1}{1+\beta}, \quad a^1_{0,0} = 0, \quad a^0_{0,1} = -\frac{1}{1+\beta},$$
$$a^1_{0,1} = -1, \quad b^0_{0,0} = b^1_{0,0} = 0, \quad b^0_{0,1} = 1. \tag{39}$$

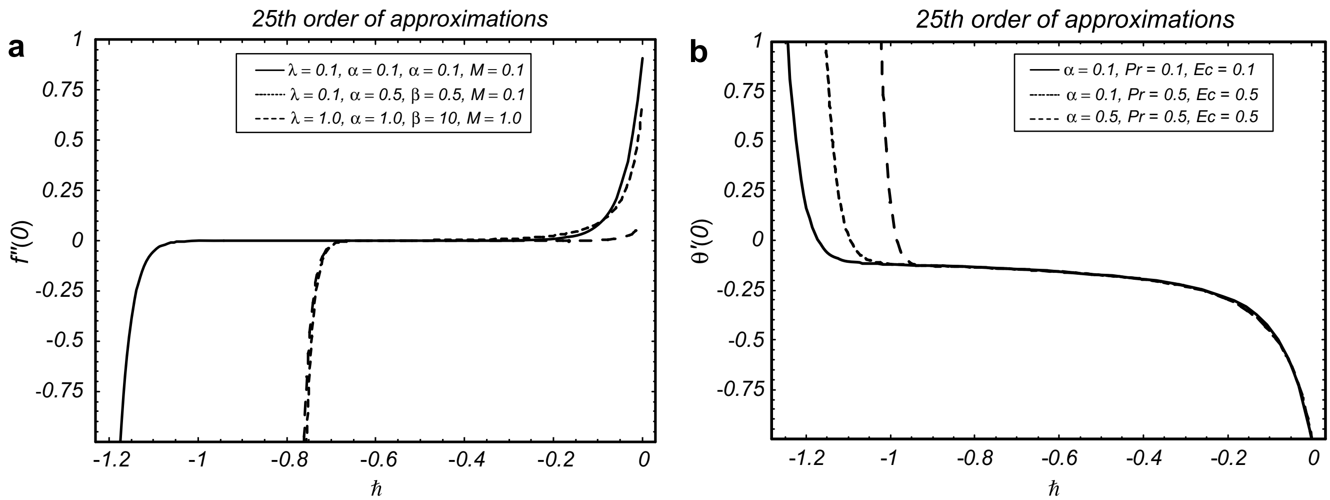

Fig. 1. $\hbar$-curves for 25th-order approximations.

4. Convergence of the HAM solutions

The explicit analytic solutions given in Eqs. 30 and 31 have the auxiliary parameters $\hbar_i$ $(i = 1 - 3)$. The convergence region and rate of approximation for the HAM solution is determined through auxiliary parameters. Due to this fact, the $\hbar$-curves for f and θ are plotted for 25th order of approximation by taking three different values of α, β, M and λ (Fig. 1). Fig. 1a shows that the range for the admissible values of $\hbar_1$ for $\lambda = 0.1$, $\alpha = 0.1$, $\beta = 0.1$ and $M = 0.1$ is $-1.1 \leqslant \hbar_1 \leqslant -0.2$. Upto these values of the parameters α, β, M and λ, the auxiliary parameters $\hbar_1 = \hbar = -0.8$ give us convergent solution. However for $\lambda = 0.1$, $\alpha = 0.5$, $\beta = 0.5$ and $M = 0.1$, the range for the admissible values of $\hbar_1$ is $-0.7 \leqslant \hbar_1 \leqslant -0.2$. For these values of the parameters, the auxiliary parameters $\hbar_1 = \hbar = -0.5$ give us convergent solution. Similarly for $\lambda = 1.0$, $\alpha = 1.0$, $\beta = 10$ and $M = 1.0$, Fig. 1b indicates that the range for the admissible values of auxiliary parameters is $\hbar_3 = -0.5$ for $\alpha = 0.1$, $Pr = 0.1$ and $Ec = 0.1$. Similarly, we can choose the value of the auxiliary parameters $\hbar_3$ for different values of parameters α, Pr and Ec. It is determined through our calculations that series given in Eqs. (30) and (31) converge in the whole region of η for different values of the physical parameters of the problem by choosing the suitable values of the auxiliary parameters $\hbar_1 = \hbar_2 = \hbar_3 = \hbar$.

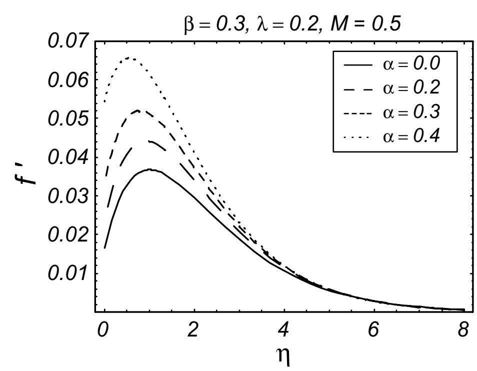

Fig. 2. Influence of α on f' at $\hbar = -0.8$.

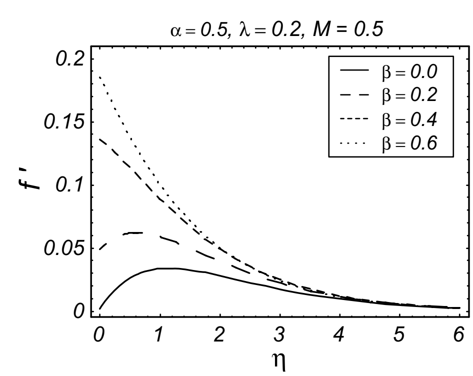

Fig. 3. Influence of β on f' at $\hbar = -0.8$.

5. Results and discussion

The purpose of this section is to see the influence of some interesting parameters on the velocity and temperature fields. In particular, attention has been focused to the second grade parameter α, the slip parameter β, porosity parameter λ and Hartman number M on the velocity. The effect of Prandtl number Pr and Eckert number Ec is also investigated on the temperature fields.

Figs. 2–5 show the effects of second grade parameter α, slip parameter β, porosity parameter λ and the Hartman number M on the velocity field f'. It is apparent from Fig. 2 that f' is an increasing function of α. It is to be noted from Fig. 3 that the velocity field f' increases as β increases. It is further observed that the effects of α and β on the velocity field is almost same in a qualitative sense. The effects of porosity parameter λ and Hartman number M on f' are sketched in Figs. 4 and 5. These two Figs. indicate that by increasing λ and M, the velocity field increases. It is worth mentioning to see that the effect of the parameter α

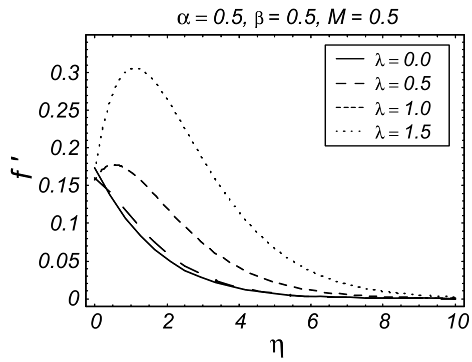

Fig. 4. Influence of λ on f' at $\hbar = -0.8$.

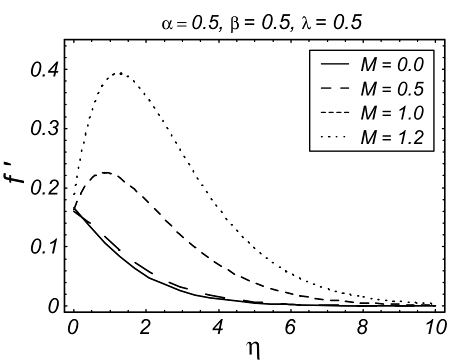

Fig. 5. Influence of M on f' at $\hbar = -0.8$.

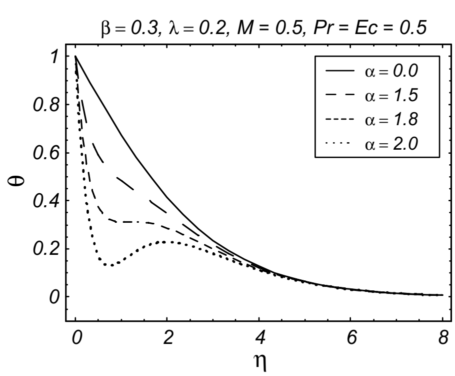

Fig. 6. Influence of α on θ at $\hbar = -0.8$.

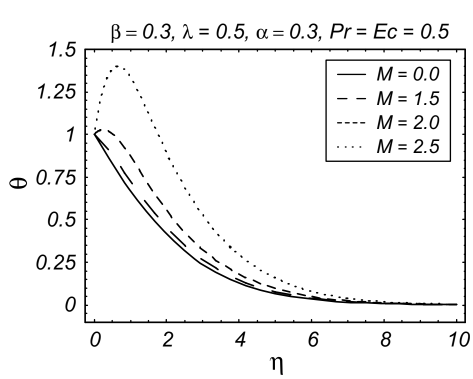

Fig. 7. Influence of M on θ at $\hbar = -0.8$.

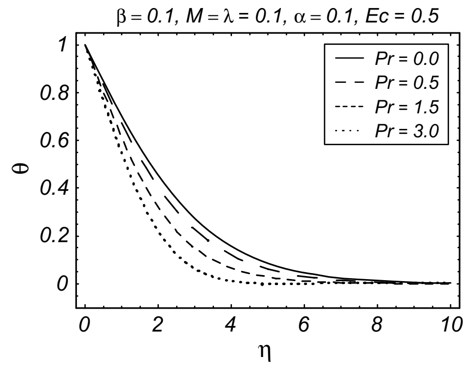

Fig. 8. Influence of Pr on θ at $\hbar = -0.8$.

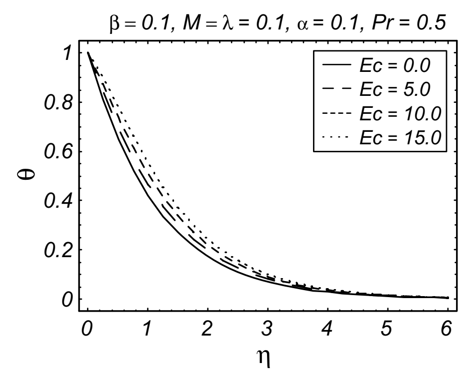

Fig. 9. Influence of Ec on θ at $\hbar = -0.8$.

on the temperature field θ is opposite to that of f' (Fig. 6). Fig. 7 depicts the effect of Hartman number M on the temperature field. It is found that behavior of M on θ is similar to that of f'. Figs. 8 and 9 show the variation in temperature field due to Prandtl and Eckert numbers respectively. It is observed that the effects of these two parameters are quite opposite. However, huge variation in the Eckert number causes little change in the velocity.

6. Concluding remarks

MHD steady flow of second grade fluid with heat transfer analysis is investigated. The flow in a porous space is due to a stretching sheet which also exhibits slip condition. The non-linear boundary condition arising through the use of slip condition is taken into account. Recurrence formulas in the series solutions for velocity and temperature are presented. The effects of various flow controlling parameters on the dimensionless velocity and temperature are analyzed. The graphical representations of these effects feature promptly. It is found that the horizontal component of velocity increases as the slip parameter increases.

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